Bifurcations of hexagonal patterns stabilized and selected with spatial perturbations in a wide-aperture laser

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Abstract. Starting from a simple hexagonal pattern stabilized and selected with the spatial perturbation method in a wide-aperture CO₂ laser, we observed the transition to a dodecagonal pattern via the doubling of the azimuthal spatial frequency by changing the cavity detuning. Further increasing the detuning induces more complicated patterns, such as double hexagons. The symmetry-breaking effect on the stabilized and selected patterns, introduced by a slight misalignment of a laser mirror, is observed. We also find that the complicated temporal instabilities of the unperturbed patterns are eliminated by the spatial perturbations. Numerical simulations, based on the Fox and Li theory for the field propagation inside the cavity, are able to reproduce the patterns with the simplest symmetry.

Pattern formation and spatiotemporal dynamics in nonlinear optics have been the subject of considerable interest during the last decade [1–3]. Wide-aperture lasers play an important role in the field of nonlinear dynamics. From the theoretical point of view, several models derived from the standard Maxwell–Bloch scheme have been proposed to interpret spatiotemporal instabilities in wide-aperture lasers. These reduced models appear in the form of the complex Ginzburg–Landau equation (CGLE) [4] or complex Swift–Hohenberg equation (CSHE) [5] which constitute the basis for studying pattern formation in physics. On the other hand, the success of different algorithms to control temporal chaos [6] has increased interest, within the field of nonlinear optics, in generalizing the control techniques to stabilize and select optical patterns and to manipulate spatiotemporal dynamics [7]. Recent experimental work by Mamaev and Saffman [8] demonstrated the successful control of optical turbulence by Fourier plane filtering in a photorefractive system.

Recently, Wang et al [9] suggested a nonfeedback method to stabilize, select and track unstable patterns based on weak spatial perturbations exerted to a control parameter μ in the form μ = μ₀ [1 + α f(r)], where μ₀ is the unperturbed control parameter, α is the amplitude of the perturbation, f(r) is the spatial perturbation function and r is the spatial coordinate (vector). Using appropriate forms of f(r), different desired target patterns can be obtained. For laser systems, the consideration of spatial perturbation can be traced back to the early stages of development in laser physics. Some preliminary observations were performed by Rigrod on a He–Ne laser [10]. In a recent experimental work [11] on a CO2 laser with a large aspect ratio, we have demonstrated that spatial perturbations produced by the insertion

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Figure 1. Experimental sequence of patterns with hexagonal symmetry, obtained by varying the cavity detuning within one free spectral range: (a) 0% FSR; (b) 28% FSR; (c) 67% FSR; (d) 79% FSR; (e) 82% FSR; (f) 87% FSR.

of thin metallic wires ($\lambda/d = 0.1$, where $\lambda$ and $d$ are the wavelength of the laser field and the diameter of the wire, respectively) in the laser cavity can stabilize and select different spatial patterns.

In this paper we present some new experimental results on spatial bifurcations of hexagonal patterns stabilized and selected with the spatial perturbation method in a wide-aperture CO$_2$ laser. Our experimental configuration consists of a Fabry–Perot cavity 700 mm long, defined by a totally reflecting flat silicon mirror and a spherical ZnSe outcoupler (region of confusion (ROC) 3 m, reflectivity 90%). The outcoupler is mounted on an electro-strictive translator to adjust the cavity detuning. The discharge column presents a high cylindrical symmetry along the laser axis since both the anode and the cathode have been prepared with annular geometry. The active medium (CO$_2$ 4.5%, He 82%, N$_2$ 13.5%), at an average pressure of 25 mbar, is pumped by means of a high-voltage DC discharge in a pyrex tube with internal diameter of 22 mm. In order to implement the spatial perturbation for the pattern stabilization and selection we insert into the cavity a mask of thin metallic wires ($d = 0.1$ mm), aligned along three directions making an angle of $60^\circ$ with each other. The mask is located at a distance of 110 mm from the outcoupler mirror. The insertion of this mask stabilizes and selects different patterns with hexagonal symmetry, starting from the simple six-lobe pattern. The spatial perturbation corresponding to the hexagonal mask can be approximated with the form $f(r) = \frac{1}{2}(e^{ik_1\cdot r} + e^{ik_4\cdot r} + e^{ik_6\cdot r} + c.c.)$ [9], where $k_i$ ($i = 1, 4, 6$) are the spatial wavevectors making an angle of $2\pi/3$ rad with each other. The magnitude of $k_i$ is $2\pi/6$ mm$^{-1}$ (6 mm is the separation between the parallel wires).

The laser output patterns were observed by means of an infrared image plate placed at a distance of 400 mm from the outcoupler mirror. In our experimental conditions the beam waist, located on the totally reflecting mirror, is $w_0 = 1.46$ mm. As a consequence, the Rayleigh range, defined as $z_R = \pi w_0^2/\lambda$, is 674 mm. This means that our observations are made in the far-field region.

By using the cavity detuning as a control parameter to explore one free spectral range (FSR), we are able to obtain the sequence of patterns shown in figure 1. Assuming the
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Figure 2. Experimental patterns with (a) eight lobes, 0% FSR and (b) ten lobes, 87% FSR obtained with a small tilt of 0.8′ of the outcoupler mirror.

Figure 3. Experimental pattern with pluri-hexagonal structure, 61% FSR, obtained with a small tilt of 1.3′ of the outcoupler mirror.

zero reference for the cavity detuning as the value corresponding to the simple six-lobe pattern of figure 1(a) \( r_0 = 4.8 \) mm is the distance of the maxima from the centre of the pattern), we first observed a smooth transition to a 12-lobe pattern (figure 1(c)). In this case we have a doubling of the azimuth spatial frequency with respect to the initial pattern and the distance of the maxima from the centre becomes \( 4r_0/3 \). Then, increasing the detuning, the 12-lobe pattern lost its stability, and we observed a double hexagon (figure 1(e)). In this case the spatial bifurcation occurs on the radial coordinate, with a halving process of the wavevector. In fact, the distances of the inner and outer peaks of figure 1(e) from the centre are \( 2r_0/3 \) and \( 4r_0/3 \), respectively. A further increase of the detuning induces a merging of the radial lobes (figure 1(f)), and finally, after one FSR, the initial condition is recovered. We note that under the conditions of our previous experiment [11], where the distance between the wires of the mask was 5 mm, variations of the cavity detuning did not lead to any bifurcation of the hexagonal pattern. It is also worth noting that the pattern sequence of figure 1 is not symmetric with respect to the central value of the control parameter (corresponding to half FSR) as predicted theoretically in [2].

A slight misalignment of the outcoupler mirror, maintaining the mask in the same position, determines a quite different scenario. In fact, it is possible to stabilize and select patterns which do not have hexagonal symmetry, in particular with eight or ten lobes as shown in figure 2, with a tilt angle of about 0.8′. We also observed more complicated structures formed by several hexagonal cells (tilt angle 1.3′) as shown in figure 3 which is typical in optical systems with a high aspect ratio.

It is important to stress that, by monitoring the local intensity, as shown in figure 4, with a small-area HgCdTe detector, the unperturbed pattern presents a complicated temporal evolution (see trace (a)) in both the alignment conditions, as already observed in this type of laser [12]. When the spatial perturbation is applied by inserting the mask, not only is the pattern stabilized and selected in spatial coordinates, but also the temporal oscillatory behaviour is removed (see trace (b)).

From the above experimental observations we can see that the two crucial control parameters which induce spatial bifurcations of the basic hexagonal pattern are the cavity detuning and the tilt angle of the outcoupler. The diffraction effect, controlled by changing the alignment of the optical cavity, can lead to a change of the symmetry of the output
Figure 4. Temporal evolution of the local laser intensity of (a) the unperturbed pattern and (b) the hexagonal pattern.

Figure 5. Numerical solutions showing the eigenmodes of the empty laser cavity: (a) and (b) perfect alignment; (c) imperfect alignment.

pattern (e.g. from a perfect hexagonal pattern to a pattern with eight lobes). In this case the symmetry of the pattern can be different from that of the mask.

Diffraction effects can be taken into account by a numerical analysis of the intracavity field propagation based on the Fox and Li algorithm [13]. Following this approach we are able to reproduce the patterns of figures 1(a), 1(c) and 2(a). In the simulation, the transverse
plane has been represented by a square grid consisting of $512 \times 512$ pixels (each pixel has a width of $50 \mu m$). The geometrical parameters of the cavity have been chosen according to the experimental configuration. Figure 5(a) shows the six-lobe pattern obtained after 500 round trips, starting with intensity and phase uniformly distributed in the transverse plane. Figure 5(b) shows the 12-lobe pattern obtained after 500 round trips, starting with uniform intensity distribution while the initial phase has been set alternately to zero and $\pi$ in 12 sectors each one $30^\circ$ wide. These numerical results show that both the hexagonal and the dodecagonal pattern are the eigenmodes of the empty laser cavity with spatial perturbation. The simulations also show that a small tilting of the outcoupler (tilt angle $\alpha = 1'$ around the horizontal axis) is responsible for the appearance of an eight-lobe pattern as shown in figure 5(c), where the initial condition is the same as in figure 5(a). However, these numerical simulations are limited to considering only the field propagation in the empty cavity. In order to explain all of our experimental results, a more complete theoretical model, which includes the mixing effect between the nonlinear action of the laser medium and the diffraction effect of the cavity, should be considered. The most suitable choice could be to combine the matter equations with the Fox and Li cavity theory. However, we believe that our experimental observations give a contribution to the description of the interplay between diffraction effects inside the cavity and the nonlinearities of the field–matter interaction of the laser.

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